



# NEURONS CAN SORT DATA EFFICIENTLY

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## TYPICAL SORTING STRATEGIES USED IN COMPUTER SCIENCE:

- Compare data and change the order of sorted elements, e.g. quicksort, heapsort, merge sort, insertion sort.
- Compute the destination positions of sorted elements using mathematical formulas, e.g. counting sort, radix sort.

## A NEW SORTING STRATEGY USING ASSOCIATIVE SPIKING NEURONS:

- Locally determine two neurons representing neighbor values and connect them to the new neuron representing an inserted new value (ASSORT-2). It is usually processed in constant or sub-linearithmic time for most of the data sets. In this strategy, no values are directly compared or counted up. Sensors compute local measures of similarities and stimulate connected neurons with different strengths. Thus, the neurons start spiking (are activated) in a different time. The activated neurons start to stimulate connected neurons in which a plasticity process can be conditionally started.



## REPRESENTATION OF VALUES AND OBJECTS USING SENSORS AND ASSOCIATIVE SPIKING NEURONS:

- This network can represent any objects  $\{O_1, \dots, O_N\}$  defined by a vector of orderable or non-orderable attribute values:  $O_n = (v_{n1}^{a_1}, \dots, v_{nK}^{a_K})$
- The attribute values (features) are represented by value sensors ( $S_{v_i}^{a_k}$ ) placed in the sensory input fields (SIF):  $F^{a_1}, \dots, F^{a_K}$ 

$$v_{min}^{a_k} = \min\{v_i^{a_k}\}$$

$$v_{max}^{a_k} = \max\{v_i^{a_k}\}$$
- The value sensors are mostly reactive to the values which they represent:
 
$$x_{v_i}^{a_k} = \begin{cases} 1 - \frac{|v_i^{a_k} - v^{a_k}|}{|v_i^{a_k}|} & \text{if } r^{a_k} > 0 \\ \frac{|v_i^{a_k}|}{|v_i^{a_k}| + |v_i^{a_k} - v^{a_k}|} & \text{if } r^{a_k} = 0 \end{cases}$$
 where  $r^{a_k} = v_{max}^{a_k} - v_{min}^{a_k}$  is a range of values of  $a_k$
- Orderable attributes allow to create extreme sensors ( $S_{min}^{a_k}, S_{max}^{a_k}$ ) and neurons ( $R_{min}^{a_k}, R_{max}^{a_k}$ ) which are specially reactive to minima and maxima:
 
$$x_{min}^{a_k} = \begin{cases} \frac{v_{max}^{a_k} - v^{a_k}}{v_{max}^{a_k} - v_{min}^{a_k}} & \text{if } r^{a_k} > 0 \\ v^{a_k} - v_{min}^{a_k} + 1 & \text{if } r^{a_k} = 0 \end{cases}$$

$$x_{max}^{a_k} = \begin{cases} \frac{v^{a_k} - v_{min}^{a_k}}{v_{max}^{a_k} - v_{min}^{a_k}} & \text{if } r^{a_k} > 0 \\ v_{max}^{a_k} - v^{a_k} + 1 & \text{if } r^{a_k} = 0 \end{cases}$$

$$t_{v_i}^{a_k} = \begin{cases} \frac{r^{a_k}}{\theta_{R_{v_i}^{a_k}}(r^{a_k} - |v_i^{a_k} - v^{a_k}|)} & \text{if } |v_i^{a_k} - v^{a_k}| < r^{a_k} \\ \infty & \text{if } |v_i^{a_k} - v^{a_k}| = r^{a_k} \\ 1 + \frac{|v_i^{a_k} - v^{a_k}|}{|v_i^{a_k}|} & \text{if } r^{a_k} = 0 \end{cases}$$
- The sensory neurons ( $R_{v_i}^{a_k}$ ) charge to the activation (spiking) threshold level in different time:
 
$$X_{R_{v_i}^{a_k}} = t_{v_i}^{a_k} \cdot x_{v_i}^{a_k} + \sum_j y_{R_{v_j}^{a_k} \cdot w_{R_{v_j}^{a_k}, R_{v_i}^{a_k}}} + \sum_n y_{O_n} \cdot w_{O_n, R_{v_i}^{a_k}} \quad w_{O_n, R_{v_i}^{a_k}} = \theta_{R_{v_i}^{a_k}} \quad \theta_{O_n} = \|R_{v_{nK}}^{a_K} \sim O_n\| = K$$

$$y_{R_{v_j}^{a_k}} = \begin{cases} 1 & \text{if } X_{R_{v_j}^{a_k}} \geq \theta_{R_{v_j}^{a_k}} \\ 0 & \text{if } X_{R_{v_j}^{a_k}} < \theta_{R_{v_j}^{a_k}} \end{cases} \quad w_{R_{v_i}^{a_k}, R_{v_j}^{a_k}} = 1 - \frac{|v_i^{a_k} - v_j^{a_k}|}{r^{a_k}} \quad y_{O_n} = \begin{cases} 1 & \text{if } X_{O_n} \geq \theta_{O_n} \\ 0 & \text{if } X_{O_n} < \theta_{O_n} \end{cases} \quad X_{O_n} = \sum_k y_{R_{v_{nK}}^{a_K}} \cdot w_{R_{v_{nK}}^{a_K}, O_n} \quad w_{R_{v_{nK}}^{a_K}, O_n} = 1$$

## CONDITIONAL PLASTICITY PROCESSES IN NEURONS WHICH CREATE AND RECONFIGURE CONNECTIONS:

- The plasticity process in the sensory input field (SIF) is started when the presented value is not yet represented by any sensor placed in it. If it is, aggregate the representation of the duplicates and do not create a new sensor and a sensory neuron.
- The plasticity process in the extreme neuron is started when it is stimulated stronger than its threshold value is:  $x_{min}^{a_k} > \theta_{R_{v_{min}}^{a_k}}$  or  $x_{max}^{a_k} > \theta_{R_{v_{max}}^{a_k}}$

## THE INTRODUCED PLASTICITY CONDITION:

- The plasticity process in the sensory neuron is started when it is less stimulated by the neighbor sensory neuron than by its sensor:

$$0 < y_{R_{v_i}^{a_k}} \cdot w_{R_{v_i}^{a_k}, R_{v_j}^{a_k}} < x_{v_j}^{a_k} - \varepsilon^{a_k}$$

- PROOF:**
- The plasticity condition is true only in the following cases:
 

Due to the sorting reason:  $\forall a_k \varepsilon^{a_k} = 0$

$$|v_i^{a_k} - v_j^{a_k}| > |v_j^{a_k} - v^{a_k}| \text{ if and only if } v_j^{a_k} < v^{a_k} < v_i^{a_k} \text{ or } v_j^{a_k} > v^{a_k} > v_i^{a_k}$$
- Substituting formulas for weights and sensory excitation to the plasticity condition we get the following:

$$0 < y_{R_{v_i}^{a_k}} \cdot w_{R_{v_i}^{a_k}, R_{v_j}^{a_k}} = 1 \cdot \left(1 - \frac{|v_i^{a_k} - v_j^{a_k}|}{r^{a_k}}\right) < 1 - \frac{|v_j^{a_k} - v^{a_k}|}{r^{a_k}} = x_{v_j}^{a_k} - \varepsilon^{a_k}$$

## CONDITIONAL PLASTICITY PROCESSES

## PLASTICITY CONDITION

