Application of S-transform to signal analysis

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Introduction

S-transform was defined by R. Stockwell. It is used, a many others, to investigate quality of energy wire, data from GPR etc. Its characteristics may be also used to examine other signals, including medical ones, like ECG. Utilizing S-transform, we can determine some of signals' properties. It is especially well-suited, because it allows to pinpoint the property in term of time and frequency. Other interesting and useful characteristics of S-transform are: independence of response's amplitude to frequency of signals, progressive resolution and information about the signal phase.

This paper presents definition and various examples of computing S-transform. The paper also covers a concept of using S-transform to investigate anomalies in signals.

Methodology

To analyse the signal, we shall use S-transform.

S-transform is defined as:

$$S_{1D}(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{i2\pi ft} dt \quad (1)$$

τ and t being time and f being frequency.

Inverse S-transform is defined as:

$$h(t) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} S_{1D}(\tau, f) d\tau \right] e^{i2\pi ft} df \quad (2)$$

Discrete S-transform is defined as:

$$S_{1D}\left[jT, \frac{n}{NT}\right] =$$
$$= \sum_{m=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} h[kT] e^{\frac{i2\pi(n+m)k}{N}} \right] e^{-\frac{2\pi^2 m^2}{n^2}} e^{\frac{i2\pi mj}{N}} \quad (3)$$

where $h[kT]$ $(k = 0, 1, 2, \ldots, N-1)$ is a series of discrete values of continues function $h(t)$. T being the sampling duration, N being the number of samples.

Inverse discrete S-transform is defined as:

$$h[kT] = \frac{1}{N^2} \sum_{n=0}^{N-1} \left[S_{1D} \left[jT, \frac{n}{NT} \right] \right] e^{\frac{i2\pi nk}{N}} \quad (4)$$

Results

In this chapter, we present images that were created as a result of S-transform computing of following signals:

- Sin
- Sin with frequency modulation
- Saw signal
- Square signal
- ECG
- ECG with distortion

S-transform has both imaginary and real parts, therefore the plots show module and angle as a function of time and frequency. HSV colours pallet was used to describe angles, which allows angles 0° and 360° to have the same colour.

Conclusion

The paper presents S-transform, which may be used for the analysis of various signals, including non-stationary ones. The analysis may be conducted on images showing module and angle (graphical representation of complex value). A database of known and anticipated signals may be created and used for image recognition, comparison (for example using artificial neural networks) and detection of unknown signals or anomalies. The possibility of using S-transform to examine ECG signal was also presented, although full implementation of this method requires further studies.

Sin

Fig. 1 Sin

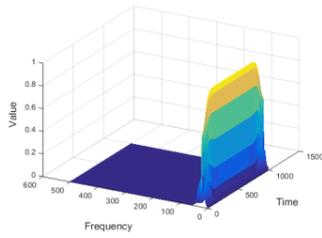Fig. 2 Flat plot of S-transform's module for sin

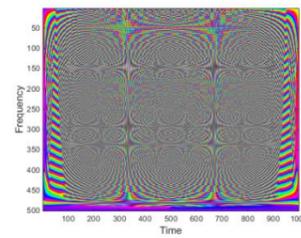Fig. 3 Spatial plot of S-transform's module for sin

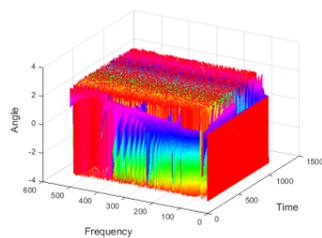Fig. 4 Flat plot of S-transform's angle for sin

Fig. 5 Spatial plot of S-transform's angle for sin
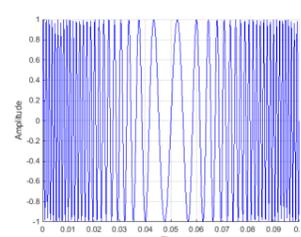
Sin with frequency modulation

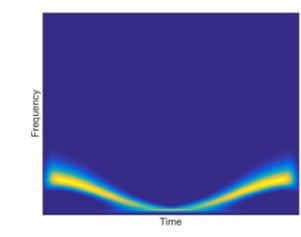Fig. 6 Sin with frequency modulation

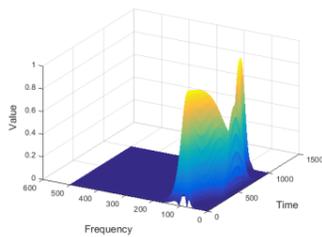Fig. 7 Flat plot of S-transform's module for sin with frequency modulation

Fig. 8 Spatial plot of S-transform's module for sin with frequency modulation
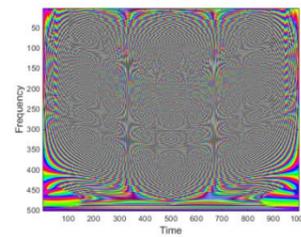

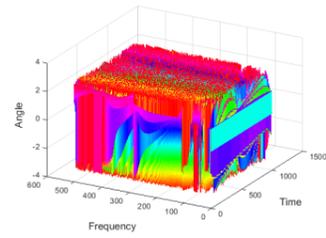Fig. 9 Flat plot of S-transform's angle for sin with frequency modulation

Fig. 10 Spatial plot of S-transform's angle for sin with frequency modulation
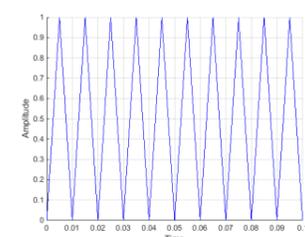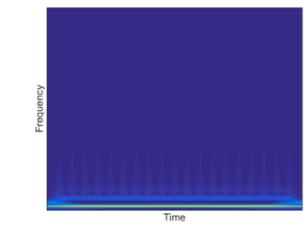
Saw signal

Fig. 11 Saw signal

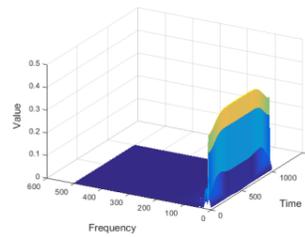Fig. 12 Flat plot of S-transform's module for saw signal

Fig. 13 Spatial plot of S-transform's module for saw signal
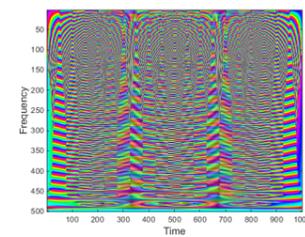

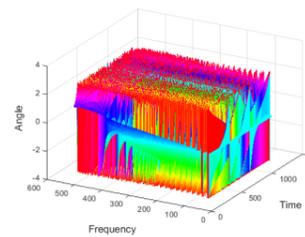Fig. 14 Flat plot of S-transform's angle for saw signal

Fig. 15 Spatial plot of S-transform's angle for saw signal
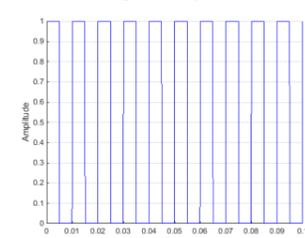
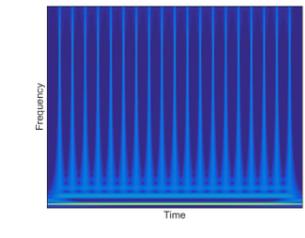### Square signal

Fig. 16 Square signal

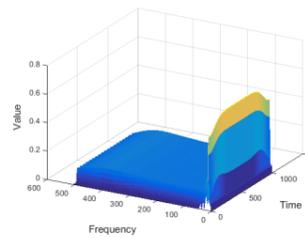Fig. 17 Flat plot of S-transform's module for square signal

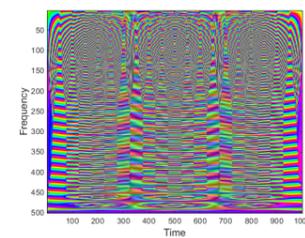Fig. 18 Spatial plot of S-transform's module for square signal

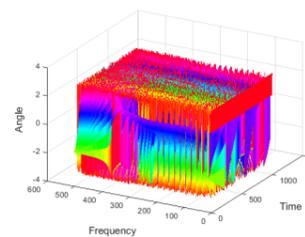Fig. 19 Flat plot of S-transform's angle for square signal

Fig. 20 Spatial plot of S-transform's angle for square signal
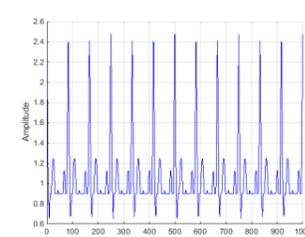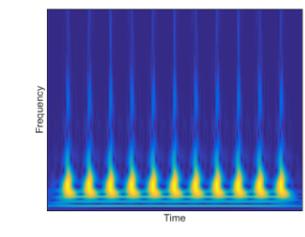
ECG

Fig. 21 ECG

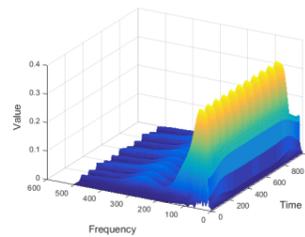Fig. 22 Flat plot of S-transform's module for ECG

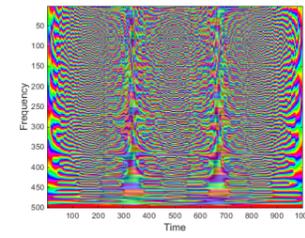Fig. 23 Spatial plot of S-transform's module for ECG

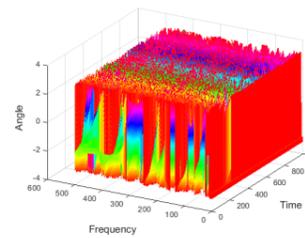Fig. 24 Flat plot of S-transform's angle for ECG

Fig. 25 Spatial plot of S-transform's angle for ECG
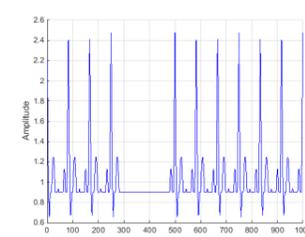
ECG with distortion
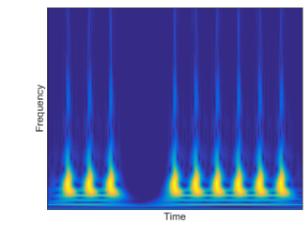

Fig. 26 ECG with distortion

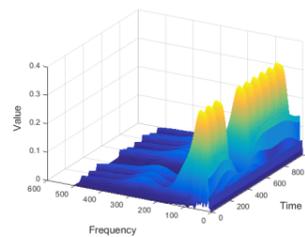Fig. 27 Flat plot of S-transform's module for ECG with distortion

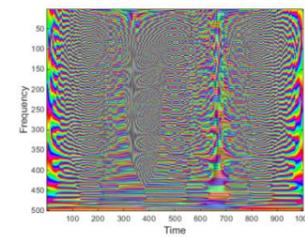Fig. 28 Spatial plot of S-transform's module for ECG with distortion

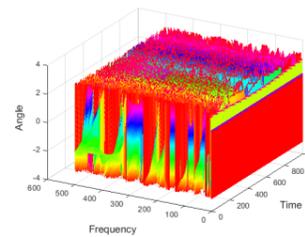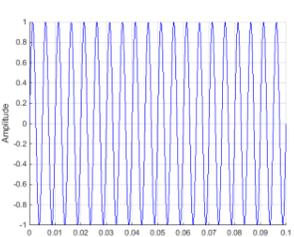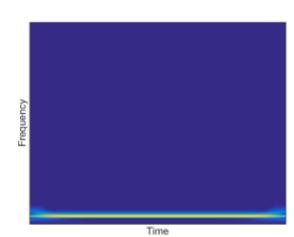Fig. 29 Flat plot of S-transform's angle for ECG with distortion

Fig. 30 Spatial plot of S-transform's angle for ECG with distortion